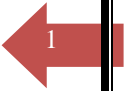


## INGLÉS II



### PRÁCTICO 3

➤ **Actividades de prelectura**

Del texto extraiga nombres de matemáticos y las fechas que se citan. Agrúpelos en las siguientes columnas.

NOMBRES	FECHAS

¿A qué rama de la matemática pertenece este texto? Justifique.

Mencione otras tres ramas de la matemática que se mencionen en el texto

➤ **Gramática**

**1)** Referencia contextual.

¿ A qué palabras o grupo de palabras hacen referencia los vocablos que aparecen en círculo? Márquelas a la manera del ejemplo.

Two finite Boolean algebras are isomorphic if (they) have the same cardinality.



2) ¿Qué función cumplen los conectores extraídos del texto. Clasifíquelos teniendo en cuenta los siguientes grupos.

- |                               |                                    |
|-------------------------------|------------------------------------|
| i.e. ( línea 2)               | a) Expresan suma o adición         |
| thus (línea 29)               | b) Contrastan conceptos            |
| also ( línea 30)              | c) Expresan relación causa- efecto |
| but (línea 32 )               | d) Introducen ejemplos             |
| however (línea 41)            |                                    |
| not only... but (línea 44-45) |                                    |

3) Subraye tres oraciones en voz pasiva y proponga un equivalente en castellano respetando estilo discursivo.

➤ **Verificando la comprensión**

Teniendo en cuenta el contenido del texto conteste las preguntas.

- Mencione las interpretaciones que Boole considera para sus identidades.
- Cite al menos dos ejemplos de álgebras de Boole que aparezcan en el texto.
- ¿En qué otras ramas o temas de la matemática tienen aplicación las álgebras de Boole?
- ¿A qué son equivalentes las álgebras de Boole?
- ¿Cuál ha sido el avance más relevante del concepto de modelo valuado Booleano?

## Introduction to Part I

The history of Boolean algebras goes back to George Boole (Boole [1854]). Boole stated a list of algebraic identities governing the "laws of thought", i.e. of classical propositional logic. The algebraic structures satisfying Boole's identities were first considered in Huntington [1904] and called Boolean algebras in Sheffer [1913].

Boole had in mind two interpretations for his identities. The first of these is the two-element Boolean algebra  $2 = \{0,1\}$ , where 0 is identified with the truth-value "false", and 1 with the truth-value "true", together with the operations corresponding to the logical ones of disjunction, conjunction and negation. The second interpretation was the "algebra of classes", where the Boolean operations were interpreted by those of union, intersection and complementation of arbitrary classes. More generally, and in a setting which avoids proper classes, every algebra of sets is a Boolean algebra. Here for any set  $X$ , an algebra of sets over  $X$  is a non-empty subfamily of the power set  $P(X)$  closed under the finitary set-theoretical operations of union, intersection and complementation with respect to  $X$ . Boole's observation amounted, in algebraic language, to saying that his identities held true under both interpretations. Interestingly enough, Boole's second example was a precursor to Cantor's set theory which began to emerge around 1874.

Only in 1921 (respectively 1936), was it proved that Boole's identities give in fact a complete axiomatization for both of his interpretations: the completeness theorem for propositional logic (Post [1921]) amounts to saying that every identity valid in the two-element Boolean algebra is derivable from Boole's axioms, and Stone's representation theorem (Stone [1936]) asserts that every Boolean algebra is isomorphic to an algebra of sets. The proofs of both results are closely connected.

Stone duality, a fundamental part of the theory of Boolean algebras, sets up an

equivalence between Boolean algebras and Boolean spaces, i.e. totally disconnected compact Hausdorff spaces. Thus, through a growing interest of topologists in Boolean spaces, Boolean algebras have a bearing on topology. They are also important in measure theory and functional analysis by way of measure spaces and Boolean algebras of projections in Banach spaces. But the main applications of Boolean algebras are still in parts of mathematics related to logic: in switching algebra, a topic not covered by our presentation, classical propositional (respectively predicate) calculus and set theory. The latter applications are based on the fact that sentences of propositional or predicate logic can be given truth values not just in the two-element Boolean algebra  $2 = \{0,1\} = \{\text{false}, \text{true}\}$  but in an arbitrary Boolean algebra. The first success of this concept of Boolean-valued model was a new and particularly lucid algebraic proof for the completeness theorems of propositional and predicate logic found by Rasiowa and Sikorski in 1950 (Rasiowa and Sikorski [1963]). The major breakthrough, however, was the observation made in 1967 by Scott, Solovay and Vopenka that Cohen's construction of generic models for independence proofs in set theory can be conceived as an instance of Boolean-valued models. Not only do complete Boolean algebras thus contribute to the understanding of models of set theory, but conversely the Boolean-valued version of Cohen's method has been applied to prove mathematical theorems on Boolean algebras by metamathematical means.

Fuente: Monk J.D. and Bonnet R., Handbook of Boolean Algebras Vol.1,2,3, Amsterdam, Northen Holland, 1989.